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A note on the minimum label spanning tree

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Abstract

We give a tight analysis of the greedy algorithm introduced by Krumke and Wirth for the minimum label spanning tree problem. The algorithm is shown to be a $(\ln(n-1)+1)$-approximation for any graph with n nodes $(n>1)$, which improves the known performance guarantee $2\ln n+1$.

Keywords: Network design; NP-hard; Approximation algorithms; Spanning tree

1. Introduction

In the minimum label spanning tree (MLST) problem, we are given a graph with labeled edges, and the goal is to find a spanning tree with the least possible number of labels. The problem was first introduced by Chang and Leu [1], motivated by applications in communication network design.

In [1], Chang and Leu proved the MLST problem was NP-hard and provided some heuristic algorithms, the quality of which was evaluated by experimental studies. In [3], Krumke and Wirth proposed a greedy algorithm, which is essentially same as the second heuristic of [1]. The algorithm is shown to have a performance guarantee of $2\ln n+1$. Moreover they proved that the problem could not be approximated within a constant factor.

We give a tight analysis of the greedy algorithm, showing that it has a performance guarantee of $\ln(n-1)+1$. On the other hand, in the original definition of MLST given in [1], the input graph is allowed to contain parallel edges. It is easy to prove that this variant has the same lower bound on approximation ratio as the minimum set cover problem. So from the results of [2], we get that the MLST problem cannot be approximated within $(1-\varepsilon)\ln(n-1)$ for any $\varepsilon>0$ unless $\mathrm{NP}\subseteq \mathrm{DTIME}(n^{\log\log n})$, which means that the performance guarantee got by us is nearly optimal.

2. Analysis of the greedy algorithm

Let $G=(V,E)$ be a connected undirected graph and $l: E\to\mathbb{N}$ be an edge labeling function. A K-labeled spanning tree T is a spanning tree of G such that the number of used labels $|\{l(e)\colon e\in E(T)\}|$ does not exceed K. A minimum label spanning tree is a K-labeled spanning tree with minimum K.

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