

Jost-Hinrich Eschenburg

# Geometry - Intuition and Concepts

Imagining, understanding, thinking  
beyond. An introduction for students



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